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RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **2607**

Mechanics 1

Monday **14 JANUARY 2002** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1

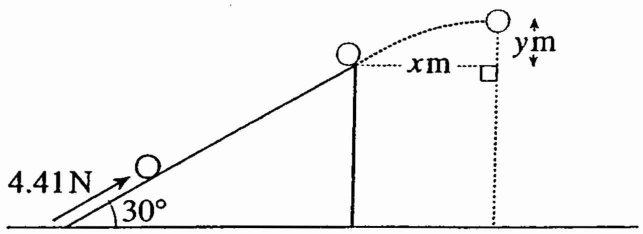


Fig. 1

In this question air resistance should be neglected.

A small ball of mass 0.6 kg is pushed up a smooth slope at 30° to the horizontal by a force of 4.41 N parallel to the slope, as shown in Fig. 1.

(i) Find the component down the slope of the ball's weight. [2]

Show that the acceleration of the ball is $2.45\,\text{m s}^{-2}$. [3]

The ball is initially at rest. The force pushes it 2.5 m to the top of the slope and then ceases to act.

(ii) Show that the speed of the ball at the top of the slope is $3.5\,\text{m s}^{-1}$.

Show that the vertical component of the velocity of the ball at the top of the slope is $1.75\,\text{m s}^{-1}$. [3]

On leaving the slope the ball becomes a projectile.

(iii) Calculate the time that elapses from the ball leaving the slope to reaching the **top** of its flight. [2]

When it is at the **top** of its flight, the horizontal and vertical distances of the ball from the top of the slope are x m and y m respectively, as shown in Fig. 1.

(iv) Show that $y = 0.156$ (correct to three significant figures) and calculate x. [4]

[Total 14]

2

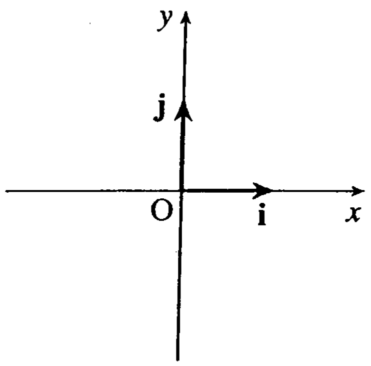


Fig. 2

In this question, the standard unit vectors **i** and **j** are in a horizontal plane in the Ox and Oy directions, as shown in Fig. 2. The units of the axes are metres.

At time t seconds, the position vector of a model boat is given by

$$\mathbf{r} = (1-2t)\mathbf{i} + \tfrac{1}{4}t^2\mathbf{j}, \qquad 0 \leqslant t \leqslant 4.$$

(i) What is the position vector of the boat when $t = 0$? [1]

(ii) The boat is at point A when $t = 0$ and at point B when $t = 4$. Calculate the distance AB. [4]

(iii) Write down an expression for the velocity of the boat at time t. [2]

(iv) Do you think that the motion of the boat as given by **r** is a sensible model for large positive values of t outside the interval $0 \leqslant t \leqslant 4$? You should give a brief reason for your answer. [1]

(v) Calculate the direction of motion of the boat when $t = 4$, giving your answer as an angle with Ox. [3]

(vi) Show that when $t = 0$, the boat is heading directly for O; [i.e. at $t = 0$, when the boat is at A, it is moving in the direction AO].

Show that the boat is moving directly away from O when $t = 1$. [4]

[Total 15]

[Turn over

3

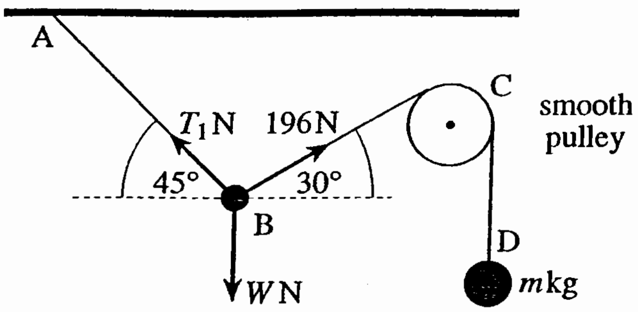


Fig. 3

Fig. 3 shows a load of weight W N at B in equilibrium. The load is attached by one light string to the ceiling at A and by a second light string that passes over a smooth pulley at C to an object of mass m kg hanging freely at D. The angles of the strings and the tensions T_1 N and 196 N acting at B are shown in Fig. 3.

(i) Write down the numerical value of the tension in string section CD, giving a reason for your answer.

By considering the equilibrium of the object at D, calculate the value of m. [3]

(ii) Calculate the value of T_1. [3]

(iii) Calculate the value of W. [3]

(iv) Calculate the magnitude of the total force exerted on the pulley at C by the string passing over it. [3]

(v) An additional mass, M kg, is now added at D. Explain why the system cannot be in equilibrium with ABC in a straight line no matter what the value of M. [3]

[Total 15]

4 **(a)** A particle travelling in a straight line at $15\,\mathrm{m\,s^{-1}}$ is brought to rest with constant deceleration in a distance of 22.5 m. Show that the deceleration takes 3 seconds. [2]

(b) A car and a bus are travelling along a straight road towards traffic lights (see Fig. 4).

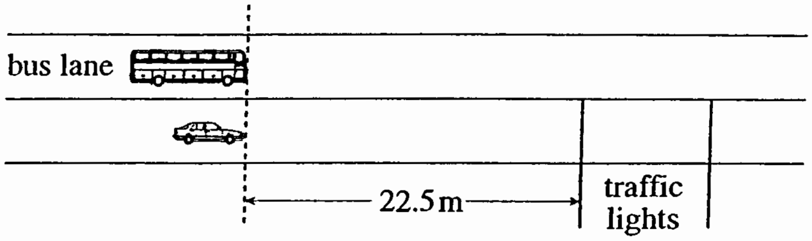


Fig. 4

The traffic lights change at time $t = 0$, where t is in seconds. At this instant the car has a speed of $15\,\mathrm{m\,s^{-1}}$.

The car then

- decelerates uniformly to rest in 22.5 m (as in part **(a)**),
- waits at the traffic lights for 7 seconds,
- accelerates uniformly up to $15\,\mathrm{m\,s^{-1}}$ in 5 seconds,
- travels at $15\,\mathrm{m\,s^{-1}}$ down the road.

(i) Sketch a v–t diagram for the motion of the car in the interval $0 \leqslant t \leqslant 20$.

Calculate the distance travelled by the car in the interval $0 \leqslant t \leqslant 15$. [6]

When $t = 0$ the car is level with a bus which is travelling at a constant speed of $20\,\mathrm{m\,s^{-1}}$ along a bus lane. The bus is not required to stop at the traffic lights and continues at this speed down the road.

(ii) Show that the bus has travelled 240 m further than the car at the time that the car again reaches $15\,\mathrm{m\,s^{-1}}$. [2]

At the instant that the car reaches its constant speed of $15\,\mathrm{m\,s^{-1}}$, the bus begins to decelerate uniformly at $0.2\,\mathrm{m\,s^{-2}}$.

(iii) It takes T seconds for the car to catch up with the bus after the bus begins to decelerate. Show that the car must travel $(240 + 20T - 0.1T^2)$ m in this time.

Hence show that T satisfies the equation $T^2 - 50T - 2400 = 0$.

Find the speed of the bus when the car catches up with it. [6]

[Total 16]

Mark Scheme

Paper 2607	Name Mechanics 1	Session January	Year 2002	Final

Solutions and mark scheme

Q 1		Mark		
(i)	$0.6 \times 9.8 \sin 30 = 2.94$ N	B1	Attempt at weight term. Accept 0.6×9.8 or $0.6g$ or $0.6\sin 30$	
		B1	2.94 correctly obtained	
	$4.41 - 2.94 = 0.6a$	M1	N2L attempted; condone mga	
	$\Rightarrow a = 2.45$ so 2.45 m s^{-2}	A1	All terms correct	
		E1	Clearly shown	5
(ii)	$v^2 = 0 + 2 \times 2.45 \times 2.5$	M1	Appropriate '*uvast*' using $a = 2.45$ Condone $u \leftrightarrow v$ and sign errors	
	$v = 3.5$ so 3.5 m s^{-1}	E1	Clearly shown from correct working	
	$v_Y = 3.5 \sin 30 = 1.75$ m s^{-1}	E1	$3.5/2 = 1.75$ is not enough	3
(iii)	Time to the top of trajectory is given by	M1	Award mark for correct value seen Condone only sign errors	
	$0 = 1.75 - 9.8t_2$ so $t_2 = 0.178571...$	A1	Must have correct working. [NB no FT as 1.75 given]	2
(iv)	For y			
	$0 = 1.75^2 - 2 \times 9.8 \times y$ or $y = 4.9{t_2}^2$	M1	Appropriate '*uvast*'; condone only sign errors	
	$y = 0.15625$ so 0.156 (3 s.f.)	E1	Must agree to 3 s.f. [SC: $u = 0$, $v = 1.75$, $a = 9.8$ and no 'from the top' SC1]	
	For x			
	$x = 3.5 \times \frac{\sqrt{3}}{2} t_2 = 0.54126...$ so 0.541 (3 s.f.)	M1	Use of **correct** horizontal component × time in the air	
		A1	Any reasonable accuracy FT **their** t_2 from (iii)	4
				Total 14

Solutions and mark scheme

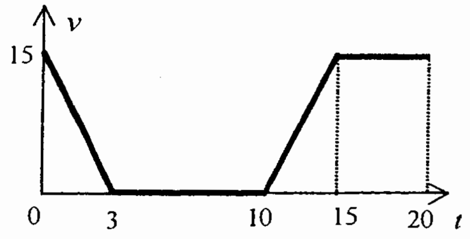
Q 2		Mark		
(i)	**i**	B1	Accept 2D and 3D cpt form. Must be a vector	1
(ii)	**OB** = (-7**i** + 4**j**)	B1	Posn vector of B	
	AB = (-7**i** + 4**j**) – **i** = -8**i** + 4**j**	B1	For **AB** or **BA** correct [The B1, B1 should be awarded if 8^2+4^2 seen]	
	AB = $\sqrt{8^2+4^2}$	M1	Use of Pythagoras (on any vector with 2 non-zero components)	
	= $4\sqrt{5}$ = 8.94427... so 8.94 m (3 s. f.)	A1	Either form cao	4
(iii)	**v** = -2**i** + 0.5t**j**	M1 A1	Differentiating; award for 1 correct cpt	2
(iv)	Predicts ever increasing value of v so no.	B1	Accept arguments involving large v or r. Some real constraint should be quoted	1
(v)	Need direction of **v**(4)	M1	Use of **v** (4)	
	i.e. –2**i** + 2**j**	A1	**v**(4) FT **their** two cpt **v**	
	Direction 135° with Ox	F1	Direction of **their** two cpt **v**. Accept 45°, -45° + 90k°, k = 0, 1, 2, 3.	3
(vi)	**v** (0) = -2**i**; (**r**(0) = **i**)	B1	For **v**(0)	
	so **v** (0) = - 2**r**(0) and heading directly for O as same direction but opposite sense	E1	Directly **towards** must be established (Equation or clear diagram sufficient)	
	v (1) = -2**i** + 0.5**j**; **r**(1) = - **i** + 0.25**j**	B1	For both **v**(1) and **r**(1)	4
	so **v** (1) = 2 **r**(1) and heading directly away from O as same direction and sense	E1	Directly **away** must be established [Need the relationship established between **v** and **r** or clear statements about parallel and sense]	Total 15

Solutions and mark scheme

Q 3		Mark		
(i)	Tension in BC same as in CD as pulley is smooth so 196 N.	B1		
	Tension in CD is weight at D so $m \times 9.8 = 196\text{N}$ $m = 20$	M1 F1	 FT their T_{CD}	 3
(ii)	Resolve → $-T_1 \cos 45 + 196 \cos 30 = 0$ or $\frac{T_1}{\sin 120} = \frac{196}{\sin 135}$	M1	Attempt at resolution involving 2 forces only in an equation. Accept sin ↔ cos (not in Lami). Condone sign errors.	
		A1	Resolution correct.	
	$T_1 = 240.049...$ so 240 N (3 s. f.)	A1	Accept any reasonable accuracy	3
(iii)	Resolve ↑ $T_1 \sin 45 + 196 \sin 30 = W$ or $\frac{W}{\sin 105} = \frac{196}{\sin 135}$	M1	Resolving vertically. All forces present. Accept sin ↔ cos (not in Lami). Condone sign errors	
		A1	All correct but FT **their** T_1 **if nec**	
	$W = 267.74...$ so 268 N (3 s. f.)	A1	Cao Accept any reasonable accuracy.	3
(iv)	**either** mag is $2 \times 196 \times \cos 30$ $= 339.48...$ so 339 N (3 s.f.)	M1 A1 A1	Attempt to resolve along line of symmetry; only condone sin ↔ cos Any reasonable accuracy	 3
	or $\sqrt{(196 + 196 \cos 60)^2 + (196 \cos 30)^2}$	B1 M1	$(196 + 196 \cos 60)$ term correct. Award if seen. Use of Pythag with 2 cpts considered	
	$= 339.48...$ so 339 N (3 s.f.)	A1	Any reasonable accuracy	
(v)	Resolve perpendicular to AC at B	M1	reference to perp component	
	There is an unbalanced cpt of W in that direction	E1	reference to an unbalanced cpt	
		E1	complete explanation	3
	or force 'triangle' is not closed	M1 E2		
	or Resolve vertically Tensions equal Complete argument	B1 E1 E1	Equation required Accept statement without reason [If string taken to be horizontal, max SC1]	
				Total 15

Paper 2607	Name Mechanics 1	Session January	Year 2002	Final

Solutions and mark scheme

Q 4		Mark		
(a)	$22.5 = \frac{(0+15)}{2}t$	M1	Use of appropriate '*uvast*' or complete method. Condone sign errors + $u \leftrightarrow v$	
	so $t = 3$	E1	Clearly shown from correct working	2
(b) (i)		B1 B1 B1 B1	Section (0,15) to (3,0) Section (**their 3**,0) to (**their 3** +7,0) Section (**their 3** +7,0) to (**their 3** +12,15) Section (**their 3** +12,15) to (**their 3** +17,15) [no labelling on axis/axes –1] [Condone $t-v$ graph; ignore $t < 0$, $t > 20$]	
	In 15 s, car travels $22.5 + 0 + \frac{(0+15)}{2} \times 5$	M1	Attempt to use both distances (May calc 22.5 again)	
	$= 60$ m	A1	cao	6
(ii)	In 15 s, bus travels $15 \times 20 = 300$ m	B1		
	Extra distance is $300 - 60 = 240$ m	E1	Clearly shown	2
(iii)	In the T s to catch up, bus travels $20T - 0.5 \times 0.2 \times T^2$	B1		
	As car is 240 m behind, total distance to travel is $240 + 20T - 0.1T^2$	E1	Clearly justified	
	distance travelled by car is $15T$	B1		
	Hence $15T = 240 + 20T - 0.1T^2$ and so $T^2 - 50T - 2400 = 0$	E1	Clearly justified	
	+ ve root is $T = 80$	M1	Solving the given equation for at least the +ve root .Award for 80 seen	
	so speed is $20 - 0.2 \times 80 = 4$ m s^{-1}	A1	cao	6
				Total 16

Examiner’s Report

Report on 2607/01 MEI Mechanics 1, January 2002

General comments

The paper seemed to allow the majority of candidates to show what they could do. The general standard of the candidates seemed considerably higher than those entered in the corresponding examination last summer and many scored high marks. Full marks were scored by many candidates on Q1 and Qs 3 and 4 were often answered very well.

There were a large number of *given* answers in this paper and it is worth re-emphasising that a greater degree of rigour is needed to establish such results than is required to find a result *not* given. Some candidates do not present a proper argument and they should know that a sea of figures, formulae or letters which contain a *given* answer will normally result in the award of zero marks unless there is evidence of a correct method being applied.

Candidates should use a sensible degree of accuracy throughout and there may be penalties for premature approximation. It is common to see a problem worked to two figure accuracy (or less) at every step but the last where three figure accuracy is claimed. Candidates are generally penalised if their working is not of sufficient accuracy to establish given results to the degree of accuracy quoted.

Q1 (Application of Newton's second law and motion of a projectile)

As already mentioned a large number of candidates scored full marks in this question.

In part (i) weaker candidates were unable to find the component of weight down the plane but were usually able to gain credit for their attempt to use Newton's second law; some candidates omitted relevant forces.

Part (ii) was usually answered correctly.

Despite being given the value of the vertical component of the velocity of the ball in part (ii), some candidates used the speed of the ball for their vertical component in part (iii). Other candidates thought that the projectile was subjected to a vertical acceleration of 2.45 m s^{-2} but many of them found the correct time.

Part (iv) was mostly well done. Common errors were to use wrong components of velocity, a non-zero horizontal acceleration and, in some cases, wrong constant acceleration formulae.

[(i) 2.94 N; (iii) 0.179 s (3 s. f.); (iv) 0.541 m (3 s. f.)]

Q2 (Vector analysis of the motion of a model boat)

The early parts of this question were answered reasonably well by many candidates but fewer did well on the last two parts.

Part (i) was usually answered correctly.

In part (ii), candidates had to find the distance AB. Many realised that the position of the boat at $t = 4$ was needed and found this correctly. It was then quite common to see an attempt at the magnitude of **OB** instead of **AB** or an attempt to find **AB** by using **AB = OA + OB**. However, it was pleasing to see that many candidates *did* find AB correctly.

Part (iii) required the derivative of the position vector; apart from the minority who attempted to use $v = u + at$, this was well done.

Relatively few candidates gave good answers to part (iv) as many of the comments made were too vague. Acceptable answers had to include the concept of the boat having to have a maximum speed or that the boat's position could not increase *without limit* in any direction. It was not enough to say that the **i and j** components became large; many candidates seemed to think that it was not acceptable to have one component of **v** (or **r**) negative and the other positive or one component much larger than the other.

In part (v), although many candidates were able to find the direction of a vector, the majority of them used **r**(4) instead of **v**(4), clearly showing that they did not understand that the direction of motion is given by the velocity vector.

Part (vi) was often poorly done. Many candidates, as in part (v), did not understand that the direction of the motion is determined by the velocity vector; others found the velocity vectors at the correct times but did not consider the position vectors. Consequently, few candidates noticed that the velocity and position vectors were parallel. Of those that did many realised that a clear diagram was sufficient to establish the easier case ($t = 0$).

[(i) **i**; (ii) 8.94 m (3 s. f.); (iii) $\mathbf{v} = -2\mathbf{i} + 0.5t\mathbf{j}$; (iv) No; (v) 135°;]

Q3 (A system of forces in equilibrium)

Compared with last summer, there was a definite improvement in the ability of candidates to resolve forces into perpendicular components and use these to form equilibrium equations. However, part (v) was rarely done well.

The calculation in part (i) was usually done correctly but many candidates did not realise or omitted to state that the pulley was smooth; many candidates stated that the tension in string section CD was the same as that in BC *because the forces were in equilibrium.*

Parts (ii) and (iii) were generally well done. Most candidates were able to write a sensible form of equilibrium equation although. as usual, a number of candidates interchanged sine with cosine and there were many sign errors. It was pleasing to note a greater number of candidates using Lami's theorem – thus making light work of these two parts of the question.

In part (iii), many candidates encountered difficulties when trying to find the resultant force exerted on the pulley. Only a minority realised that this could be done simply by resolving along the line of action of the resultant force (the bisector of angle BCD). The more common method (finding the horizontal and vertical components of the resultant force and then calculating its magnitude using Pythagoras' theorem) usually gave a wrong answer because of the omission of one part of the vertical component. Some candidates thought that simply adding the two components together would produce the required force.

Only the most able candidates were able to make progress in the final part. Many stated (incorrectly) that, *the tensions in the string would be different which would make equilibrium impossible*. Those who made progress usually did so by drawing a clear diagram and then either explaining clearly how the forces along or normal to ABC led to a resultant force or forming the equilibrium equations and demonstrating the same.

[(i) 196 N, 20; (ii) 240 (3 s. f.); (iii) 268 (3 s. f.); 339 N (3 s. f.)]

Q4 (Kinematics of a car and a bus)

Most candidates obtained the first ten marks for this question even when they had scored few marks elsewhere in the paper.

Part (a) was usually correctly answered although there were some sign errors. Some candidates "lost" this error either by conveniently deleting the minus sign or by making a second sign error – in either case marks were deducted.

Part (b)(i) required a *v-t* diagram to be sketched and this was usually done correctly for full marks. Candidates then had to use the diagram (or an *otherwise* method) to find the distance travelled in the first 15 seconds of motion. This was also usually done correctly except that some candidates misread the question and calculated the distance travelled during the first 20 seconds.

Part (ii) was found to be straightforward and there were very few incorrect answers.

The given result $240 + 20T - 0.1T^2$ in part (iii) was not always established properly for full credit. It was common to see solutions with no explanation that started with statements such as $-240 = 20T - 0.1T^2$ or to see $s = s_0 + ut + \frac{1}{2}at^2$ quoted and then the given answer without any further justification.

Few candidates obtained the quadratic equation, usually because they failed to appreciate that the printed expression needed to be equated to $15T$.

Many candidates correctly solved the given equation and were then usually able to find the speed of the bus.

[(b)(i) 60 m; (iii) 4 m s^{-1};]